Mathematics

Progression in Written Maths Calculations

Mathematics teaching should:-

- provide children with a balance of exploration, acquisition, consolidation and application
- ensure that children experience the excitement of learning mathematics
- direct and steer children to explore, identify and use rules, patterns and properties and model this process
- build in frequent short and sharp periods of practice and consolidation
- engage with children's thinking, giving sufficient time for dialogue and discussion and space to think
- demonstrate the correct use of mathematical vocabulary, language and symbols, images, diagrams and models as tools to support and extend thinking
- give opportunities for children to use and apply their learning
- teach children how to evaluate solutions and analyse methods and understand why some methods are more efficient than others
- review children's learning with them

Progression in all Four Operations

Progression in calculation should include:

A range of mental strategies to be used as a first resort (even after written methods have been introduced and embedded).
An ability to understand the relationships between the four operations.
An ability to explain, describe and record their methods.
An ability to estimate and check whether the answer is correct.
An ability to solve a wide range of problems.
An ability to choose and use the most appropriate method of calculation (whether that is mental, jottings, written or use of a calculator).
An ability to take the initiative to return to an earlier method that a child may be more confident with.

Informal recording

- It is important to allow children to explore solving calculations using paper and pencil methods. This enables children to organise their thinking in a way meaningful to them and supports them in explaining how they solved a calculation.

- Children record their informal jottings on whiteboards. These are valuable written methods.
- It is important that children do not abandon jottings and mental methods once other pencil and paper procedures are introduced. Children will always be encouraged to look at a calculation / problem and then decide on the best method.

It is vital that the children have the opportunity for a lot of practical work as this will prevent many errors and misconceptions occurring.

Use plenty of practical, hands on practice with Cuisenaire, Dienes apparatus, multibase/Base 10 materials, number lines and 100 squares in order to introduce:-

- Adding with larger numbers particularly to show what happens to the units when they exceed 10, the tens when they exceed 10 etc… When the children understand this and can use and apply it the teacher can show this alongside the written method to explain what is happening, why it is happening and how it is happening.
- Subtraction – a vital step not to be missed when introducing the expanded form of decomposition. Children will carry out the exchange and see that the value of the whole number has not changed but the value of each digit has changed – redistributed.
- Multiplication – vital for sets of, lots of groups of - has 3 meanings
    - Repeated addition
    - Describing an array
    - Making something a number of times longer/bigger/larger etc..

  Valuable to show 3x4 is 3 rows of 4 and 4x3 is 4 rows of 3.
- Division – vital for practical work - 3 meanings
    - repeated subtraction
    - sharing equally
    - put things into equal groups

<u>**Written recording**</u>

Although this booklet focuses on pencil and paper procedures, it is important to recognize that the ability to calculate mentally lies at the heart of the numeracy framework. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin and develop new ideas. Written methods, therefore, both help children to extend and clarify their thinking.

The children need to learn and refine written methods of recording. They also need to use and apply what they have learnt to problem solving tasks. When children have learnt a new method of recording a written calculation it is vital they practice, reinforce, consolidate, use and apply it to mathematical learning and NOT simply move onto the next method of recording.

Every year the findings from our own school mathematics analysis show the key area for development as Using and Applying. Reports show children can manipulate written methods and carry out calculations but are not as successful choosing the correct operation and subsequent written calculation to solve a word problem.

Remind children they are carrying out calculations and NOT sums. Sum means addition.

<u>Progression in values of numbers and digits</u>

In all four operations the progression in value of numbers and digits is all follows:-

- U +/-/x/÷ U up to 10
- U +/-/x/÷ U up to 20
- TU +/-/x/÷ U up to 20
- TU +/-/x/÷ U within 50, 100 etc…
- TU +/-/x/÷ TU
- HTU +/-/x/÷ U
- HTU +/-/x/÷ TU
- HTU +/-/x/÷ HTU
- ThHTU +/-/x/÷ U
- ThHTU +/-/x/÷ TU
- ThHTU +/-/x/÷ HTU
- ThHTU +/-/x/÷ ThHTU

Etc…

- Children need to be taught the appropriate vocabulary associated with each operation at each stage (listed at the beginning of section for each of the four operations).
    - Children need to have mental strategies for each operation. They will draw upon these to support them with written methods.
    - It is important steps are taught, practiced, reinforced and consolidated before new methods are taught.
    - Children need to be able to apply what they have learnt to problem solving.
    - Throughout their mathematics learning, children are creating their own 'toolkit' of strategies, facts and methods.

- At all stages it is important that children are taught inverse operations ie addition / subtraction and multiplication /division . Children should be encouraged to estimate what the answer to a calculation should be and then to always check their answer (possibly by using the inverse operation) when they have finished.

Teaching a new method

- When teaching a new method of writing down a calculation it is important to work with numbers the children can deal with mentally so they are focusing upon one thing only – how to 'work' the written method.
- It is useful when moving from one written method to a more refined/standardised way of recording to show the children both methods alongside each other. This enables them to understand how the methods both produce the correct answer.
- Children need to understand that they will build up a toolkit of many methods and that all these methods work. As they become successful in using and applying their mathematics, they pick the most efficient method with which they are confident.
- It is useful for the teacher, when modelling a new written method, to record on the board, and out loud, their thinking so children hear and see what is happening, why it is happening and how it is happening. This is valuable when refining column addition with carrying figures, decomposition and standardised short division.

Progression in written methods for ADDITION

Vocabulary to be taught

equals, is the same as
add, addition, more, plus, increase
sum, total, altogether
score
double, near double
how many more to make…?
how many more
how much more

First Steps in Addition

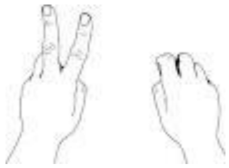
There is not a lot of written recording but the beginning of strategies to help with the concept.

Practical activities_________recorded by photographs. Real life situations…How many more do we need? (Milk, fruit, boys and girls)
Using objects to count two sets, finding "How many altogether?"
Using fingers to show how many and to add two sets together.
The children begin to record combining sets and "adding one more" in pictorial representation.
Using number lines with pegs

Use dominoes to count 2 sets separately and then how many altogether or a ladybird with different numbers of counters on its wings.
Children are beginning to use numerals to record the total or an adult will scribe.
Numerals are then written under a pictorial recording of each set and a total at the end.
Finally a number sentence 5 + 5 = 10 shown as a formal recording.
The children will have problems to solve where they find the answer by counting out, using fingers or objects and the only numeral they will write is the answer.
They will then use objects such as a tower of 5 cubes and break it into 2 parts to write all the stories of 5 and so will practise the formal recording they have been used to reading.
Fingers used to show doubles to 10.
Number lines and washing lines used to count up.
Put bigger number in head and count on with fingers showing the smaller number.
Children need to be able to count on from any number, combining two groups. They may develop ways of recording calculations using pictures or apparatus, such as Cuisenaire.

Children will recognise that addition can be done in any order, eg: 10 = 3 + 7
10 = 7 + 3

Children will begin to know addition facts to 10, they will begin to recognise how they can use their fingers to help them with number bond facts. Teach how the addition facts are linked to subtraction facts – inverse operation.

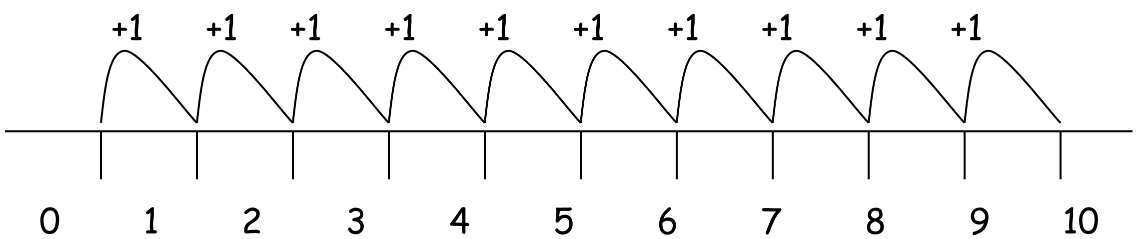


2 fingers up. How many are down?

Introduce the children to train tracks as a first number line to use for addition.

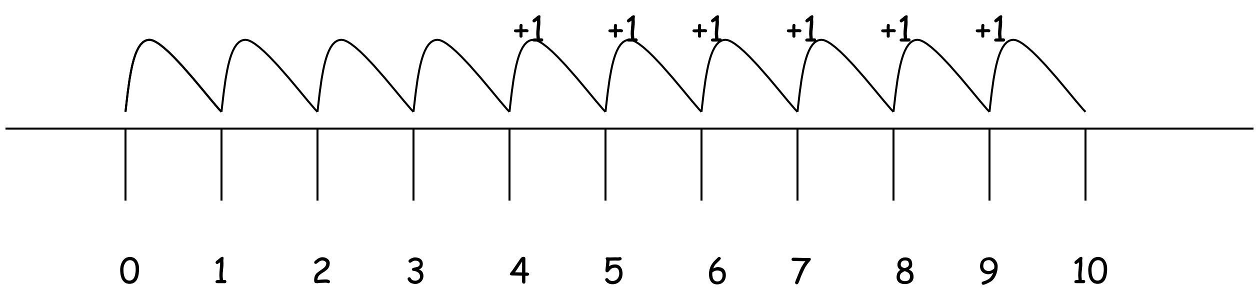
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

Children will be able to count on in ones on a numbered line.



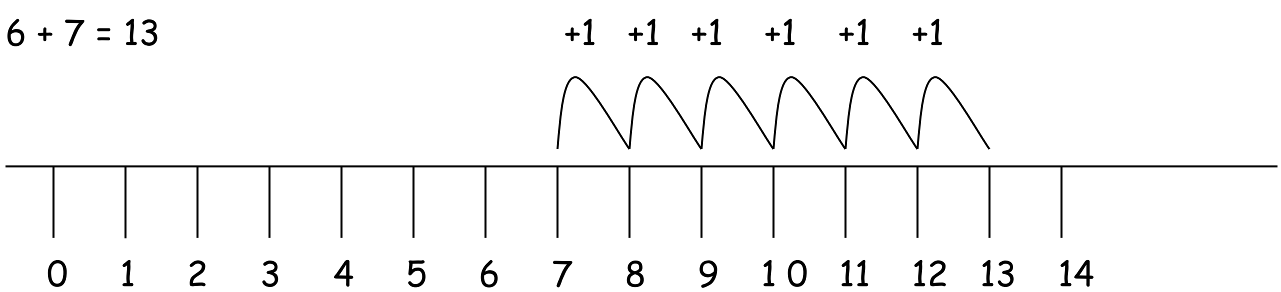
They will be able to count on from zero, and once they become confident with counting, they will be able to count on from the first number.

4 + 6 = 10



Children will be able to use more efficient jumps, starting with the larger number and counting on in ones. They will be able to record their calculation as a number sequence.

6 + 7 = 13

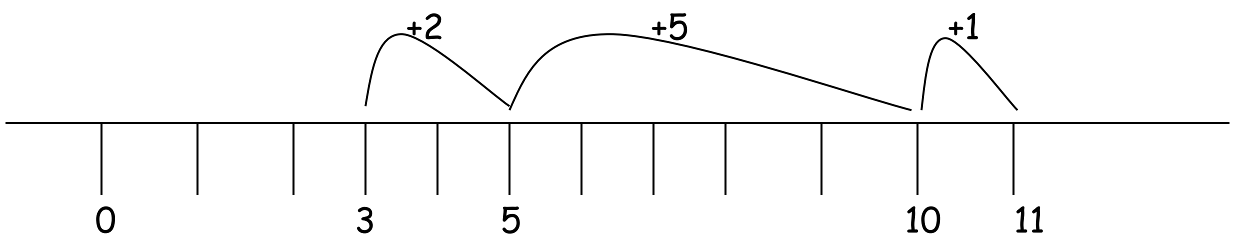


Children are able to count on using a marked empty number line, then by drawing their own number line.

They begin to jump various amounts, applying number bond knowledge to help them 'bridge' to the next 5 or 10. Use a prepared number line initially.

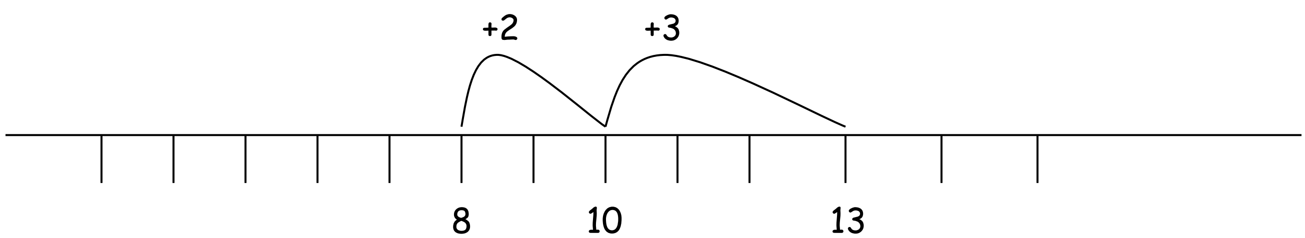
<u>Bridging (jump) to 5</u>

3 + 8 = 11



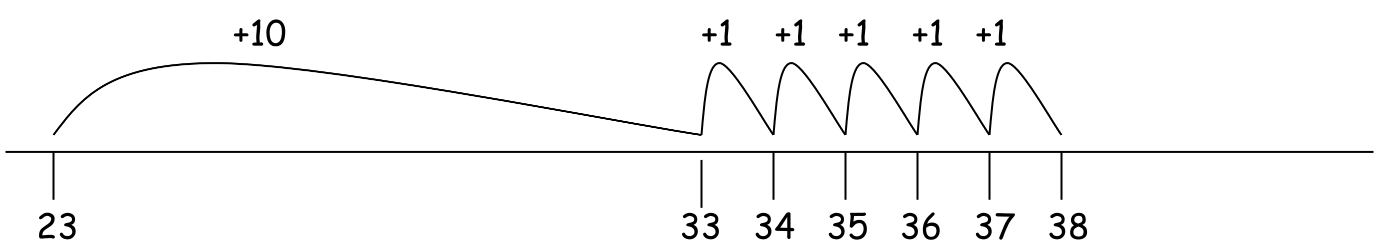
<u>Bridging (jump) to 10</u>

8 + 5 = 13



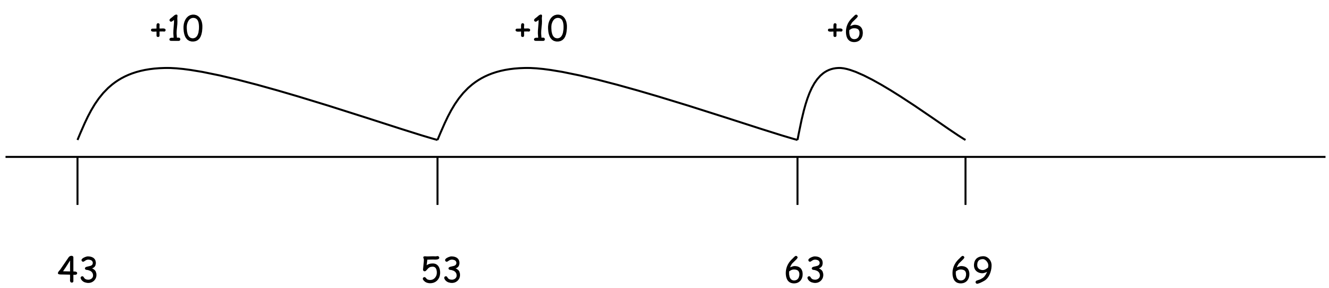
They use their knowledge of number patterns to count on in different sized steps. The children will be able to partition two digit numbers; they will be able to count on in tens and multiples of ten. Children will be encouraged to count on from the largest integer.

23 +14 = 37
23 +10 + 4 = 37 (partition 14 into 10 and 4)

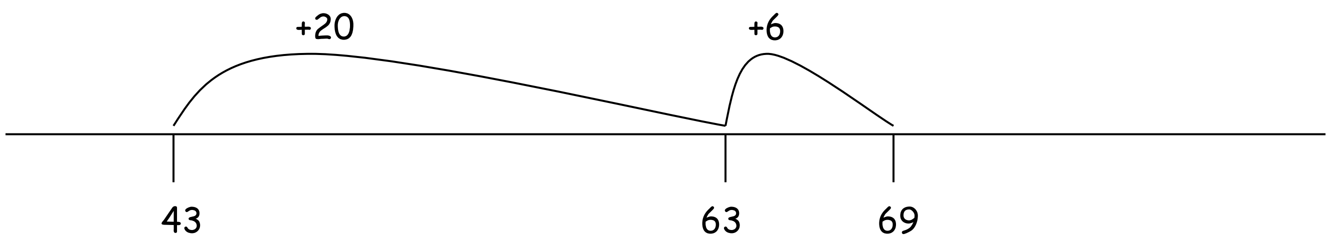


The children count on in tens and then in ones.

26 + 43 = 69   43 + 10 + 10 + 6 = 69 (partition 26 into 10, 10 and 6)

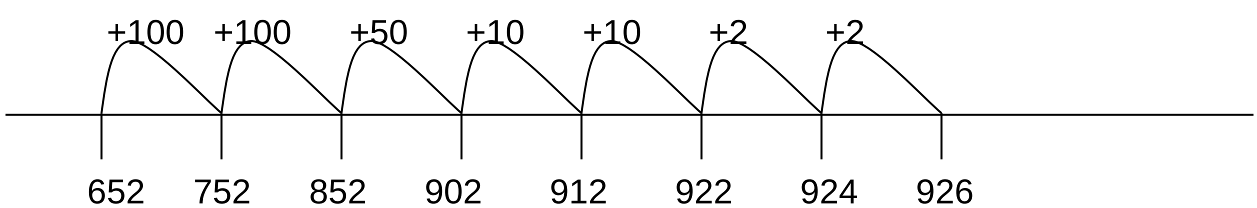


OR



They will progress onto using number lines to add larger numbers more efficiently.

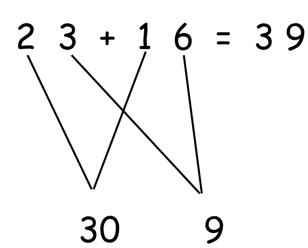
652 + 274 = 926



Record calculations as number sentences and number line method.

Children will use their knowledge of place value to partition numbers into tens and units which are then added separately then recombined. The partitioning method will be used before the formal column method at each stage of the progression.

23 + 16 = 20 + 10 = 30
3 + 6 = 9
= 30 + 9 = 39



The children will be able to solve addition using the Partitioning Column Method and will understand the importance of lining up the place value digits. This can closely be linked to horizontal methods of mental recording as shown below.
They partition the number and add each place value separately, always starting with the least significant digits (eg: the units)

```
  T U
  2 3        20 + 3
+ 1 6        10 + 6
-----       -------
  3 9        30 + 9
-----       -------
```

They 'carry' into the tens when the units are more than 9.

The carried tens are recorded under the answer box.

```
  T U
  4 7  ---->  40 + 7    Both parts could be recorded initially if
+ 3 4  ---->  30 + 4    the children need to.
-----       -------
  8 1  <----  70 + 11
-----       -------
  1
```

The children 'carry' into the hundreds when the tens are more than 90.

```
  H T U
  2 4 7        200 + 40 + 7
+   8 2              80 + 2
-------       -------------
  3 2 9        200 + 120 + 9
-------       -------------
  1
```

The children will be able to solve additions using the Column Method and will understand the importance of lining up the place value digits.
They understand the place value of each digit, adding from the least significant digit first.

```
  T U
  4 2
+ 3 4
-----
  7 6
-----
```

The children 'carry' underneath (below the line) when the digits and figures go over their value.

	T	U
	4	7
+	3	4
	8	1
	1	

	Th	H	T	U
	1	0	4	7
+		9	8	4
	2	0	3	1
	1	1	1	

The children will be using the Compact Addition method to solve addition of decimal numbers.

They apply the same rules to 'carrying' to decimal numbers.

	T	U	$\frac{1}{10}$
	2	4 .	6
+	3	4 .	7
	5	9 .	3
		1	

	T	U	$\frac{1}{10}$	$\frac{1}{100}$
	2	4 .	6	4
+	3	4 .	7	9
	5	9 .	4	3
		1	1	

Adding fractions

With the same denominators:

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

With convertion to common denominator:

$$\frac{2}{7} + \frac{6}{14} = \frac{4}{14} + \frac{6}{14} = \frac{10}{14}$$

Answer can then be rewritten in lowest form.

With mixed whole numbers and fractions

$$2\frac{2}{7} + 6\frac{4}{7} = 8\frac{6}{7}$$

Mixed fractions with need for common denominators (working out could be shown.)

$$2\frac{3}{7} + 6\frac{4}{21} = 2\frac{9\ (3\times3)}{21\ (3\times7)} + 6\frac{4}{21} = 8\frac{13}{21}$$

Progression in written methods for <u>SUBTRACTION</u>

<u>Vocabulary to be taught</u>

equals, is the same as
subtract, subtraction, take (away), minus, decrease
leave, how many are left/left over?
difference between
half, halve
how many more/fewer is… than…?
how much more/less is…?

<u>First steps in subtraction</u>

Children will have had a lot of practical experience using the language of subtraction through counting games and rhymes eg
Use of number rhymes.
Use rhymes using numbers within 5 e.g. Five currant buns, five little men in a flying saucer and five little speckled frogs.
Use rhyme with numbers within 10 e.g. Ten green bottles, ten fat sausages and ten in the bed. Using practical resources children will see visual subtraction.

Children need to understand the difference between:-

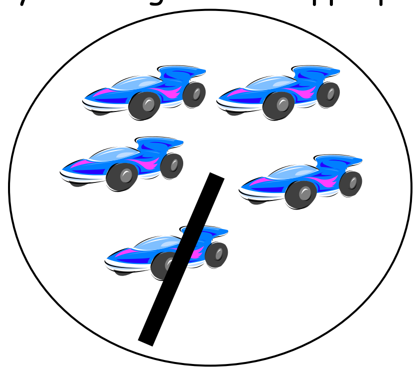
- Take away – children can remove and count what is left.
- Find the difference between – we encourage children to count on to find what is between 2 numbers.

Progression in Written Calculations - Subtraction

Children working with a hoop and cars – I have five cars and one goes into the garage, how many are left?

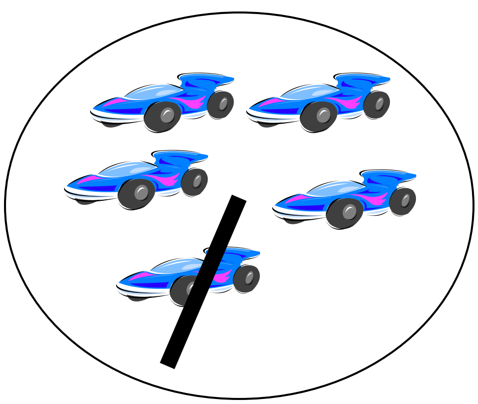
Children can take the appropriate number away – photographic recording would show this process as a visual way of recording.

This would transfer to pictorial recording where a child would have a set of objects and would record by crossing out the appropriate number.



Children move on to compare two sets of objects to understand the concept of more and less. Identify which set has more or less using physical objects. Ask ' what is the difference?'. As children compare like for like and remove one from each set they will see how many more one set had etc…

As children are introduced to the symbol for subtraction this can be used with pictures and numbers in a sentence, using the = sign.



$$5 - 1 = 4$$

Children progress to recording their written calculations using the digits, subtraction symbol and the = sign. As with addition this would be recorded in different ways. Teach the link between addition and subtraction ie inverse operations.

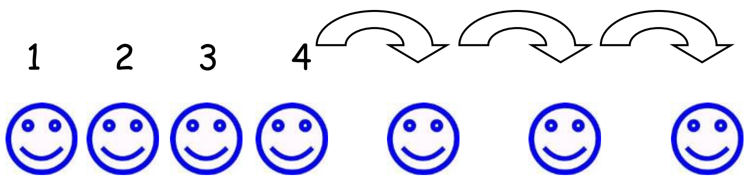
5 – 1 = 4

4 = 5 – 1 etc….

When the numbers have a difference which can not be calculated mentally, children begin to use number lines in order to record the difference between 2 numbers.
It is important that children see the relationship between find the difference and take away. They follow the same principle as we encourage children to count on for the smallest number up to the largest in both cases

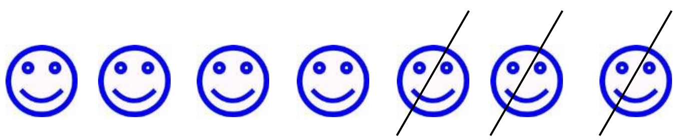
Before children can move onto the methods for subtraction they need to be able to count reliably including one to one correspondence.

Children will be able to count up or back from any number.



How many more will I need to make 7?

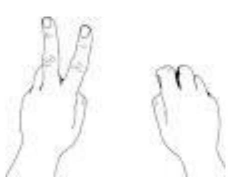
Children will understand subtraction as taking away.



Remove some objects and count.

Children will begin to know the INVERSE relationship of number facts up to 10.

7 + 3 = 10   3 + 7 = 10
10 – 3 = 7   10 – 7 = 3



2 fingers up. How many are down?

8 fingers down. How many are up?

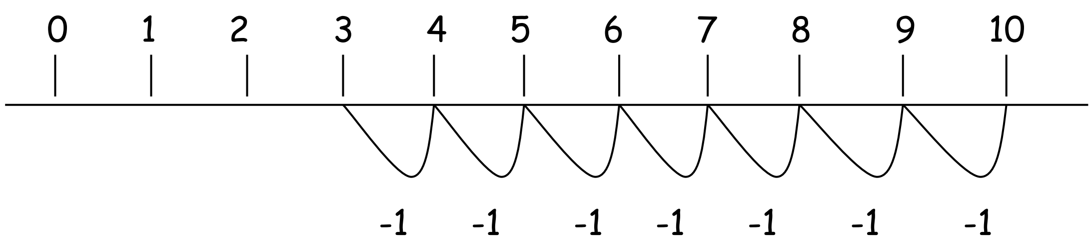
The children will be supported with these concepts through singing nursery rhymes and develop ways of recording calculations using pictures or using apparatus.

Introduce the children to train tracks as a first number line to use for subtraction.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

The children will be able to count back in ones on a numbered line. For subtraction place the numbers on the top of the line and the jumps underneath (inverse to addition).

10 – 7 = 3

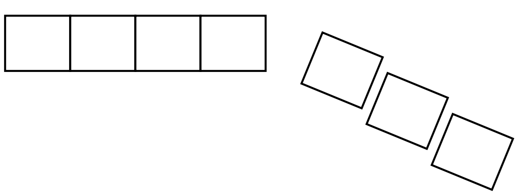


They will be able to record their calculation as a number sequence.
Children will be able to understand the concept of subtraction as:

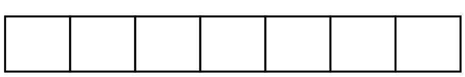
~ Taking away
~ Finding the difference
~ Counting up
~ Counting back

At all stages the children will be taught that subtraction is the inverse of addition and vice versa.

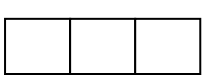
7 – 3 = 4



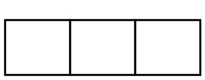
One quantity with 3 removed



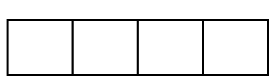
Two quantities. 4 more make them the same.



(I have 3, how many more do I need to make 7?)



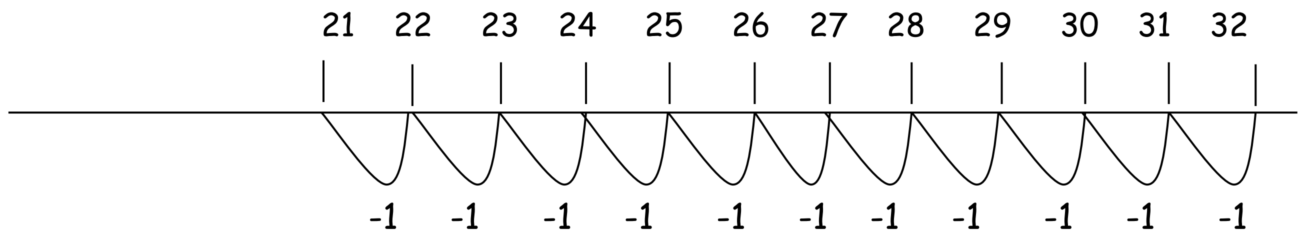
Three



four more to make 7.

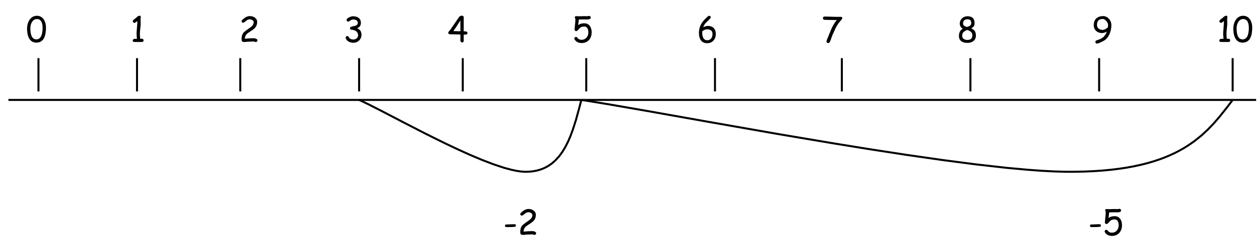
Children are able to count back using marked unlabelled number lines, then by drawing their own number line.

32 – 11 = 21



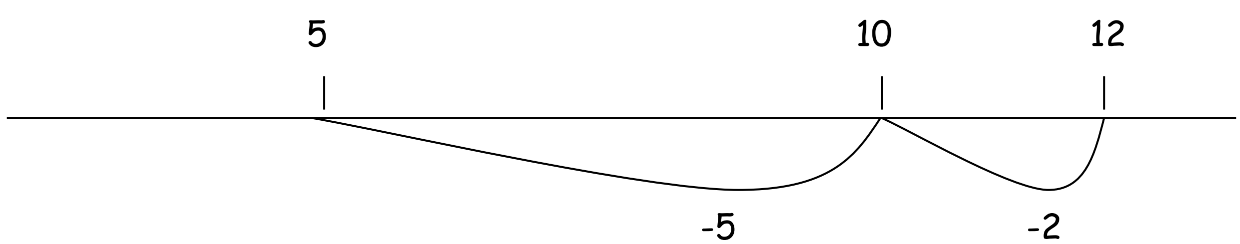
They begin to use jumps of various sizes, applying number bond knowledge to help them 'bridge' to 5 or 10.

10 – 7 = 3



(more efficient jumps on labelled number line)

12 – 7 = 5



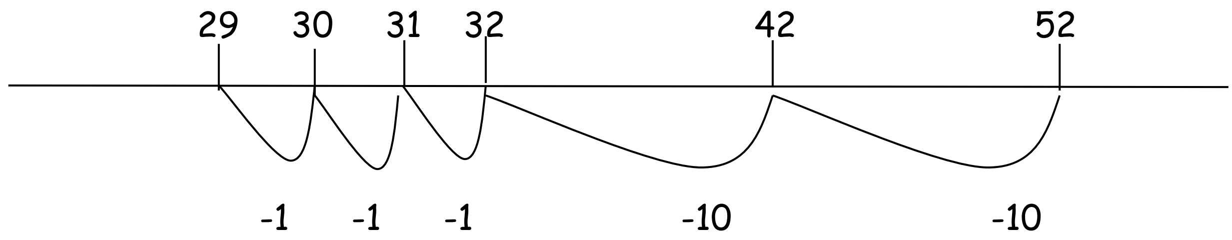
(more efficient jumps on empty number line)

They use their knowledge of number patterns to count back in different sized jumps.

The children will be able to partition two digit numbers. They will be able to count back in tens and multiples of tens.

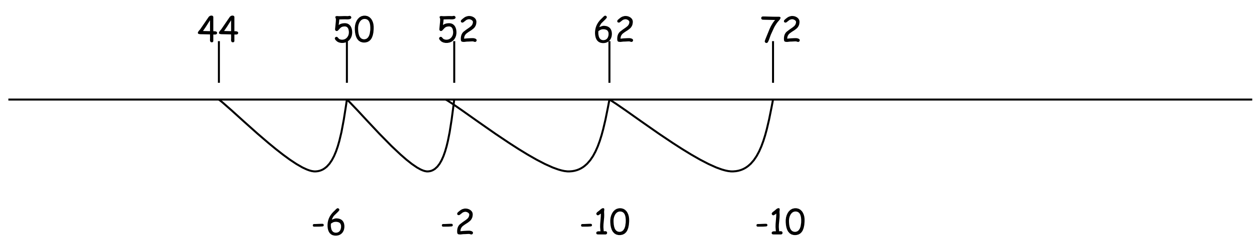
52 – 23 = 29

They count back in tens first then in ones.



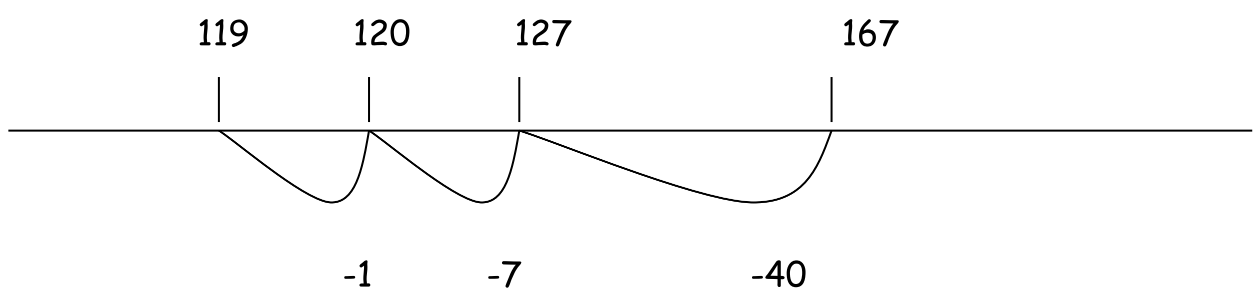
72 – 28 = 44

They count back in tens, then in more efficient jumps.



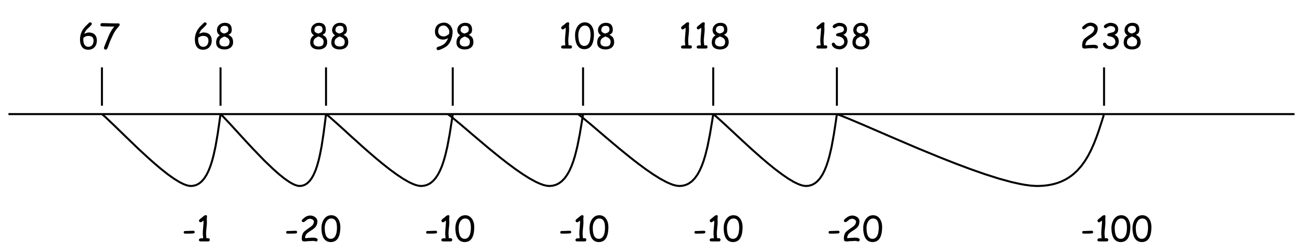
167 – 48 = 119

They count back in more efficient jumps of multiples of 10.



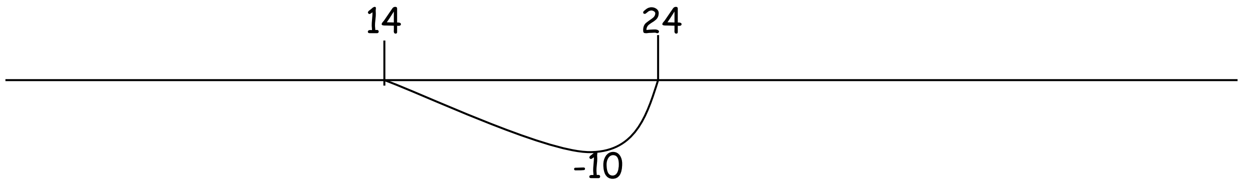
They use number lines to subtract larger numbers more effectively.

238 – 171 = 67

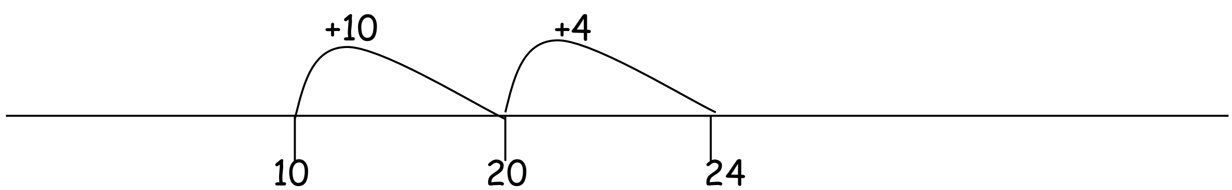


The children will understand the link between subtraction as finding the difference and taking away.

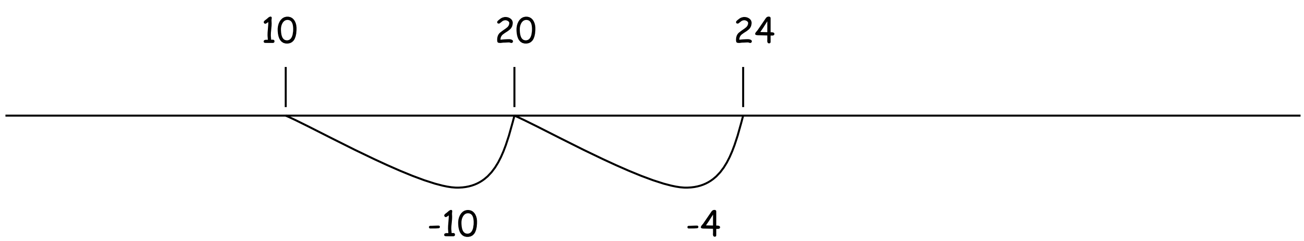
24 – 10 = 14 (take away)



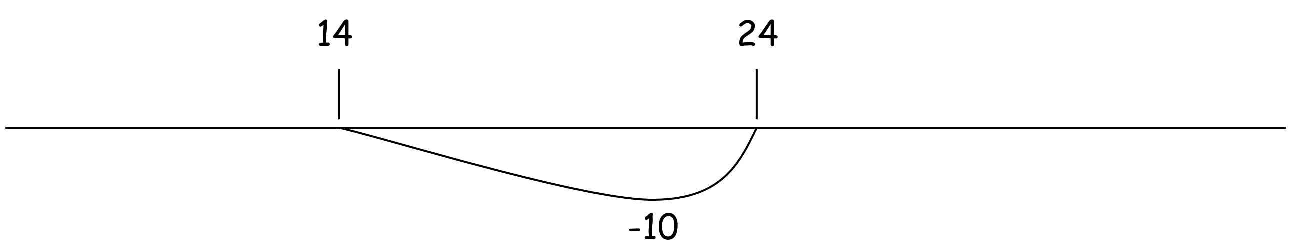
10 + ? = 24 (find the difference)



24 – 14 = 10 (take away)



24 - ? = 14 (find the difference)



They will use the column method (known as decomposition) to partition the number and subtract each place value separately, starting with the least significant digit (the unit). It is vital that they understand the importance of keeping the digits lined up.

Initially, no exchanging over. Eg: the units or tens of the number being subtracted is smaller than the starting number.

68 – 32 = 36

	T	U
	6	8
-	3	2
	3	6

6 0	+		8
- 3 0	+	-	2
3 0	+		6

Next, they will learn how to exchange from the tens to the units. They need to recognize when the starting number's units have less than the number being subtracted. When this is the case, they need to 'exchange' 10 from the tens into the units, in order to be able to subtract the numbers. It is vital the children understand the tens have to come over to the units as a whole ten. Base 10 (Dienes) apparatus helps the children to visualize what happens and how the exchange takes place.

334 – 217 = 117

	H	T	U
	3	3	4
-	2	1	7
	1	1	7

		20		14
3 0 0	+	~~3 0~~	+	~~4~~
2 0 0	+	1 0	+	7
1 0 0		1 0		7

Exchanging from hundred to tens should be introduced next. The children need to understand that 100s are exchanged as a whole 100. Be careful as some children try to just take 10 from the 100.

537 – 274 = 263

	H	T	U
	5	3	7
-	2	7	4
	2	6	3

400		130		
~~5 0~~ 0	+	~~3 0~~	+	7
2 0 0	+	7 0	+	4
2 0 0		6 0		3

Next, the children need to exchange from tens to units, and hundreds to tens.

521 – 376 = 145

```
 H  T  U                     10     11             400        110
 5  2  1        5 0 0 + 2̶0̶ + 1̶                 5̶0̶0̶    +  1̶0̶  +  1 1
-3  7  6        3 0 0 + 7 0 + 6                3 0 0   +  7 0  +   6
 1  4  5                                       1 0 0      4 0      5
```

Children will be able to subtract using the Compact Decomposition Method and understand the importance of lining each digit up.

<u>Exchanging tens to units.</u> They understand the place value of each digit, subtracting from the least significant digit first.

334 – 217 = 117

```
   H  T  U
      2  1
   3  3̶  4
 - 2  1  7
   1  1  7
```

<u>Exchanging hundreds to tens.</u> They know to look carefully at the numbers and identify which parts (HTU) need exchanging, before beginning. They still need to start with the least significant figure first.

537 – 274 = 263

```
   H  T  U
  4  1
   5̶  3  7
 - 2  7  4
   2  6  3
```

<u>Exchanging tens to units and hundreds to tens.</u>

534 – 378 = 156

```
   H  T  U
  4  12  1
   5̶  3̶  4
 - 3  7  8
   1  5  6
```

<u>Exchanging when there are no tens.</u>

Often a common misconception is that 10 can be carried from 100 into the units, or 100 can be carried into the units. It is really important the children understand they have to carry 100 into the tens, and then carry the ten into the units ~ it takes two steps to get the units ready for subtraction to take place.

504 – 247 = 257

```
   H  T  U
  4  9 1  1
  5̶  0̶   4
- 2  4   7
  2  5   7
```

The children will be using the Decomposition Subtraction method to solve subtraction of decimal numbers. They use the same rules for exchanging to decimal numbers.

```
  T  U  1/10

     7    1
  3  8̶ .  2
- 2  4 .  7
  1  3 .  5
```

```
  T  U  1/10  1/100

     3   11    1
  6  4̶ .  2̶    1
+ 2  1 .  7    2
  4  2 .  4    9
```

Subtracting fractions

With the same denominators:

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$

With convertion to common denominator:

$$\frac{6}{7} - \frac{6}{14} = \frac{12}{14} - \frac{6}{14} = \frac{6}{14}$$

Answer can then be rewritten in lowest form.

With mixed whole numbers and fractions

$$8\frac{6}{7} - 6\frac{4}{7} = 2\frac{2}{7}$$

Mixed fractions with need for common denominators

$$6\frac{3}{7} - 2\frac{4}{21} = 6\frac{9\,(3\times3)}{21\,(3\times7)} - 2\frac{4}{21} = 4\frac{5}{21}$$

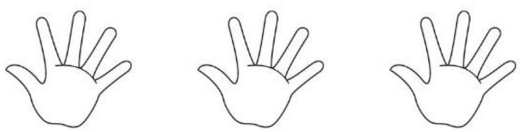
Progression in written methods for <u>MULTIPLICATION</u>

Vocabulary to be taught

equals, is the same as
lots of, groups of
times, multiply, multiplication, multiplied by
multiple of, product
once, twice, three times… ten times…
times as (big, long, wide… and so on)
repeated addition
array, row, column
double, halve

Children will experience equal groups of objects and will be able to count in 2s, 5s and 10s. Practical problem solving activities, involving equal sets or groups and use of apparatus such as Cuisenaire, will help children to visualise the grouping of numbers and support counting on as repeated addition.

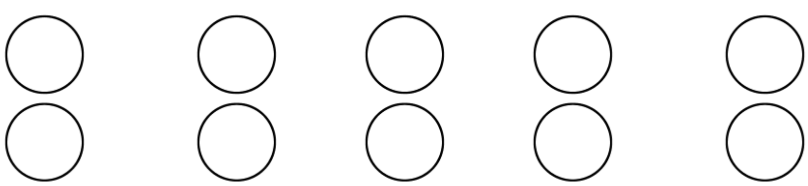
2 4 6



5 10 15

10 20 30

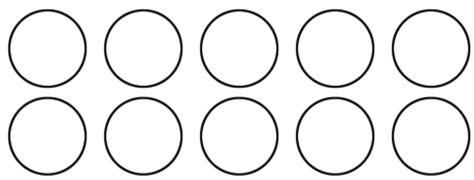
Children will begin seeing these groups as ‘lots of’ ‘groups of’, eg:



5 lots of 2 = 10

Counting on in 2s, 5s and 10s (brought in through different topics).
Chanting.

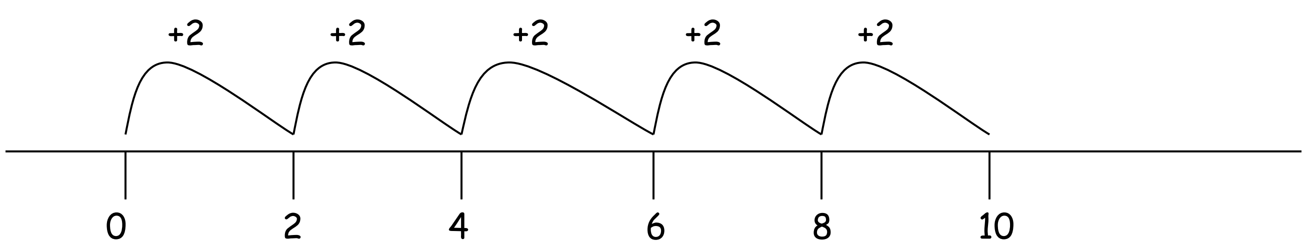
The children will understand that two equal groups of objects or numbers is 'double'. The vocabulary 'double' or 'doubling' should be used alongside 'multiplied by 2,' 'times 2,' '2 lots of,' 'two groups of' 'two sets of'



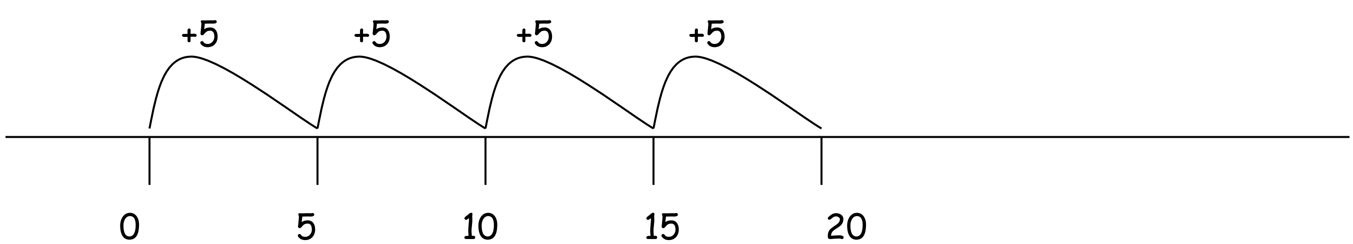
5 + 5 = 10
5 X 2 = 10

Children will be able to represent jumps of 2, 5 and 10 on a number track or a numbered number line and relate it to the concept of repeated addition.

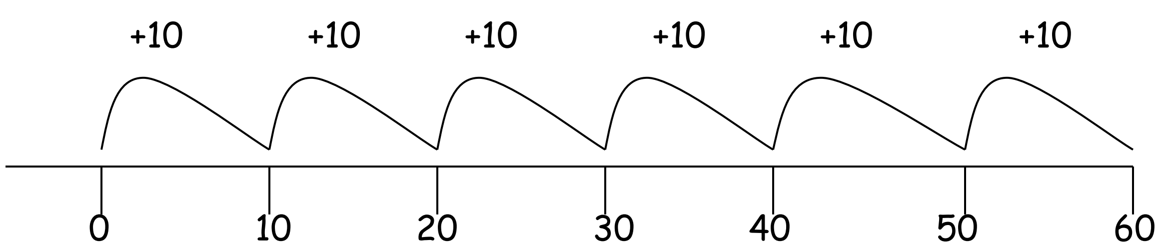
2	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**



The children will begin to see the relationship between addition and multiplication, so
2 + 2 + 2 + 2 + 2 = 10 is the same as 5 X 2 = 10 (X being lots of, groups of).



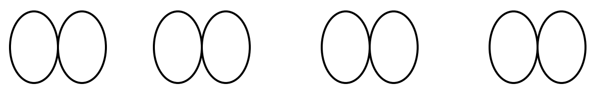
5 + 5 + 5 + 5 = 20 is the same as 4 x 5 = 20



10 + 10 + 10 + 10 + 10 + 10 = 60 is the same as 6 X 10 = 60

Pupils may still need the support of apparatus Cuisenaire Rods or Dienes apparatus to help to visualise the concept of grouping and repeated addition.

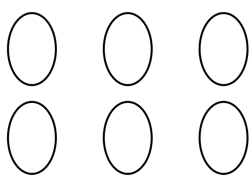
Pupils will begin to see that is doesn't matter which way round a multiplication calculation is done, that it always has the same answer. This is shown in dot arrays. This is called the 'commutative' method of multiplication.



2 + 2 + 2 + 2 = 4 x 2 = 8

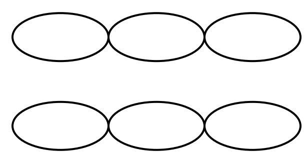
4 + 4 = 2 x 4 = 8

The use of apparatus or number lines will help to visualise this concept, they will see how repeated additions will still come to the same total. Introduce the children to arrays.



2 + 2 + 2 = 6

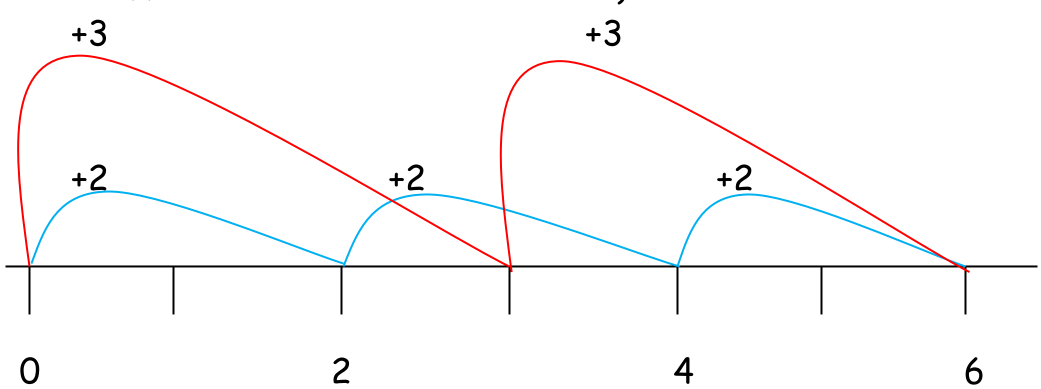
3 x 2 = 6



3 + 3 =6

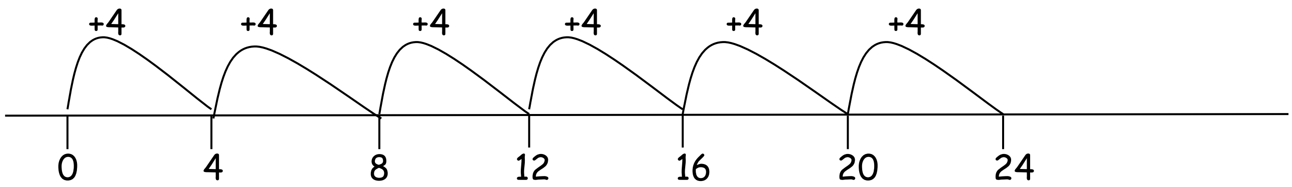
2 x 3 = 6

(Use two different colours to demonstrate.)



Children will be able to draw blank number lines, know that for multiplication they are to start from zero, and they will understand the commutative nature of multiplication and will select the number they feel more confident in grouping by for repeated addition. They will be able to visualize in groups of dots or arrays.

Eg: 6 x 4 = 24



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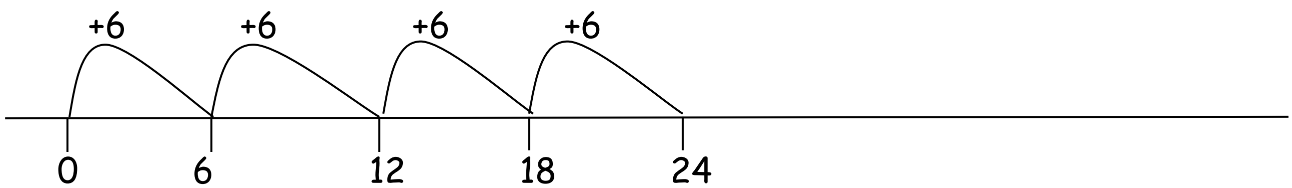
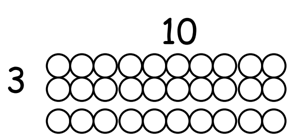


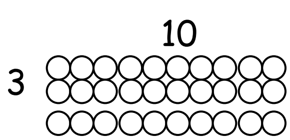
Pupils will be able to use times tables knowledge to multiply larger numbers, by partitioning 2 digit numbers into tens and units. Dot arrays and Dienes (base 10) apparatus will help children to visualise this..

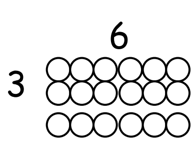
26 x 4 = (10 + 10 + 6) x 3

10 x 3 = 30
10 x 3 = 30
6 x 3 = 18
78

and







Chanting times tables eg 1 x 2 = 2
2 x 2 = 4
3 x 2 = 6 etc

Teach related facts eg $1 \times 2 = 2$

$2 \times 1 = 2$

$2 \div 1 = 2$

$2 \div 2 = 1$

Emphasise difference between tables and counting up in multiples of numbers.

At this stage children will be able to partition any 2 digit and 3 digit numbers and multiply by a multiple of 10.
They will know 2x, 5x and 10x table and be able to calculate 3x, 4x, 6x, 7x, 8x and 9x.

Pupils will be able to multiply by a multiple of 10, using their knowledge of place value (they will know that when a number is multiplied by 10, the number gets 10x larger and the digits move one place value to the left). Dienes apparatus may be used to support pupils when visualising this.

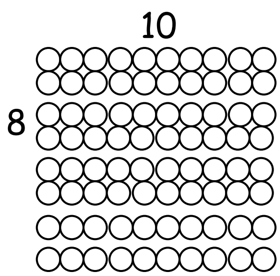
$$\begin{aligned} 50 \times 6 &= (5 \times 6) \times 10 \\ &= 30 \times 10 \\ &= 300 \end{aligned}$$

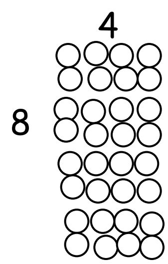
The children will then be introduced to the grid method to calculate TU x U. They will learn to partition the two digit number into tens and units and then multiply each part separately. They will finally add the numbers together to get the final answer.

14 x 8

X	8
10	8 0
4	+ 3 2
	1 1 2

Dienes apparatus will show this visually, or, by using dot arrays showing an array of 14x8 into 10x8 and 4x8, children will understand that this will give them the same answer.





Once confident with using grid method, pupils will progress onto multiplying larger two digit numbers and then three digit numbers. They need to continue applying their mental times tables knowledge to simplify numbers (ie: 40x 6 = (4x6)x10), and multiply numbers by ten.

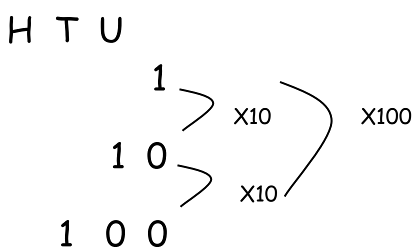
27 x 6

X	6
20	1 2 0
7	+ 4 2
	1 6 2

134 x 4

X	4
100	4 0 0
30	1 2 0
4	+ 1 6
	5 3 6

Children will know how to move the digits one place to left when multiplying by 10, and two places when multiplying by 100. This is particularly important in preparation for multiplying decimals.



500 x 6 = (5 x 6) x 100
= 30 x 100
= 3000

Pupils will be secure with place value and be confident when multiplying numbers like 60x40. They will simplify to 6x4, then understand that they made the both numbers 10x smaller, therefore there answer will need to be made 10x larger twice (100x larger).

60x40 = (6x4) x100
= 24 x 100
= 2400

They will then progress onto using the grid method to multiply TU x TU.

34 x 27

X	20	7
30	600	210
4	80	28

```
   H T U
   6 0 0
   2 1 0
     8 0
 +   2 8
   9 1 8
   1
```

Pupils will be able to use the grid method to multiply HTU x TU.

234 x 27

X	20	7
200	4000	1400
30	600	210
4	80	28

```
  Th H T U
   4 0 0 0
   1 4 0 0
     6 0 0
     2 1 0
       8 0
 +     2 8
   6 3 1 8
   1 1
```

The children will be able to multiply any decimal number by 10, 100 and 1000. They will understand the place value of decimals and know how to move the digits 1 place to left when x10, 2 places to left when x 100 and 3 places to left when x1000.

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$	
		0	.	0	1	
						X10
		0	.	1	0	x100, x1000
						X10
		1	.	0	0	
						X10
	1	0	.	0	0	
						X10
1	0	0	.	0	0	

They will simplify numbers and adjust when multiplying by decimals, eg:

$6 \times 0.16 = (6 \times 16) \div 100$
$= 96 \div 100$
$= 0.96$

Children will then move on to using grid method to multiply decimal numbers by U.

6.7 x 8

X	8
6	48
0.7	5.6
	53.6

T	U	.	$\frac{1}{10}$
	6	.	7
x			8
5	3	.	6
	5		

Pupils should move on to consolidating and refining multiplication methods and applying them to 2-step problems.

Pupils will be able to use the grid method to multiply decimal numbers by TU, leading onto the formal method.

4.24 x 23

X	4	0.2	0.04
20	80	4.0	0.8
3	12	0.6	0.12

```
        1    1
T U .  10  100
8 0 .  0   0
  4 .  0   0
  0 .  8   0
1 2 .  0   0
  0 .  6   0
  0 .  1   2
--------------
9 7 .  5   2
--------------
  1
```

```
   4.24
X    23
  -----
  12.72
  84.80
  -----
  97.52
  -----
   1
```

18.34 x 7

X	10	8	0.3	0.04
7	70	56	2.1	0.28

```
          1    1
H T U .  10  100
  7 0 .  0    0
  5 6 .  0    0
    2 .  1    0
    0 .  2    8
----------------
1 2 8 .  3    8
```

Some pupils may be proficient in using the grid method and fully understand place value that they will be able to move onto a more efficient method of multiplication (the column method).

The first step for developing the column method is the expanded column method, which shows how it links to the grid method.

24 x 56 =

```
  Th H T U
       2 4
X      5 6
----------
       2 4    4x6    ~ multiply by 6 initially
     1 2 0    20x6
     2 0 0    4x50   ~ multiply by 50
   1 0 0 0    20x50
----------
   1 3 4 4           ~ to complete answer, all numbers need adding together
```

Once competent with the above method, pupils can move onto the compact column method of multiplication.

	Th	H	T	U	
			2	4	
X			5	6	
			2		
		1	4	4	~ begin with 4x6=24, put 4 in U, carry 2 into tens. Then 20x6 (add on 2) = 14
		2			
	1	2	0	0	~ multiplying by 50, begin by putting place holder in U. Then 4x5=20, put 0 in tens carry 2 into hundreds. Then 2x5=10 (add on 2) = 12
	1	3	4	4	

Multiplying fractions by a whole number

Multiply the numerator by the whole number and the denominator remains unchanged.

$$2 \times \frac{2}{5} = \frac{4}{5}$$

Multiplying fractions by another fraction

$$\frac{1}{2} \times \frac{2}{5}$$

Multiply the numerators (top numbers)

$$\frac{1}{2} \times \frac{2}{5} = \frac{1 \times 2}{} = \frac{2}{}$$

Multiply the denominators (bottom numbers)

$$\frac{1}{2} \times \frac{2}{5} = \frac{1 \times 2}{2 \times 5} = \frac{2}{10}$$

Simplify the fraction:

$$\frac{2}{10} = \frac{1}{5}$$

Progression in written methods for <u>DIVISION</u>

<u>Vocabulary for division</u>

equals, is the same as

Share out	Equal groups of
Half	Remainder
Halve	factor
Divide	quotient
Divided by	Inverse and reverse
Divided into	
Divisible by	

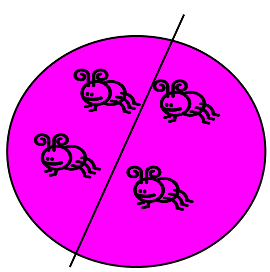
Divisor   12 ÷ 3   3 is the divisor

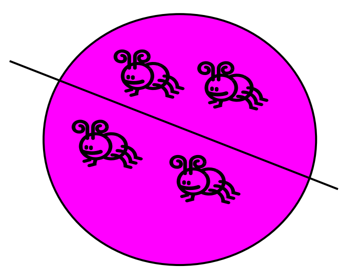
÷ 2 means 'how many 2s go into….?'   (this links back to the multiplication)

'Share by' should never be used. It is mathematically incorrect and has never been included in the vocabulary list for mathematics.

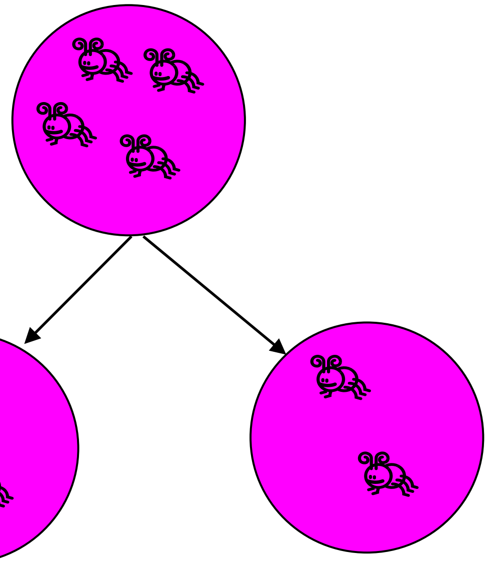
The following stages are the result of practical work first before any attempts at recording in a written or pictorial form.

Children can initially record how they have shared a set into equal groups by using 'strings' practically and drawn lines to record.





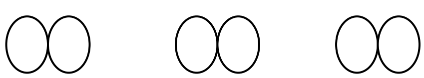
Children can progress to using a set and sharing it into equal groups. This would be carried out practically then explained in written format.



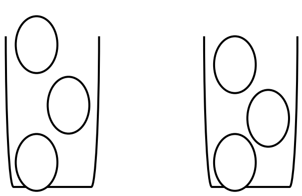
Children will experience counting in 2s, 5s and 10s and will be given many opportunities to group objects in these amounts. The focus will be on 'groups' and 'grouping' and at this age, these words become key for the children to recognise this means division / to divide.

Children will use counters, objects or Cuisenaire to divide into equal groups of two or two equal groups. The children will begin to recognise how times table facts can help them to see the inverse relationship between division and multiplication.

Introduce the concept of remainder.



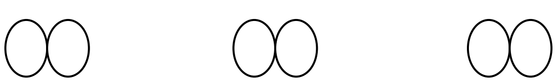
6 in 3 groups of 2's (grouping)



6 into 2 equal groups of 3 (sharing)

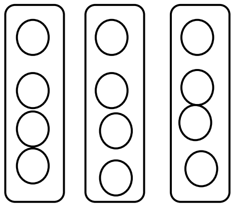
6 ÷ 2 = 3 and 3 x 2 = 6

(6 divided into groups of 2 = 2 groups) (3 groups of, lots of 2 = 6)



Children will move on to using dot arrays to solve division problems using the vocabulary 'sharing' and 'grouping'. This will be initially taught practically, but once children become more confident, they will be able to draw jottings and dot arrays to support.

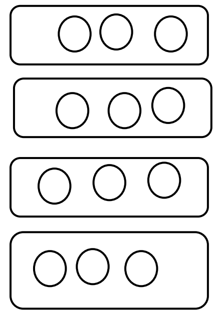
Grouping



3 groups of 4 = 12

3 x 4 = 12

12 in groups of 4
=3 groups

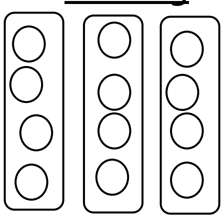


4 groups of 3 = 12

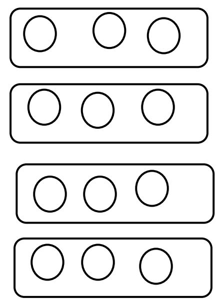
4 x 3 =12

12 in groups of 3
= 4 groups

Sharing



12 shared into groups of 4
= 3 groups
12 ÷ 4 = 3



12 shared into groups of 3
= 4 groups
12 ÷ 3 = 4

'The use of the symbol ÷ for division can also cause problems. The statement 12 ÷ 3 arises out of two practical situations, either as "share 12 objects equally among 3 people" or as "how many groups of 3 are there in 12?" Both are symbolised by 12 ÷ 3 which needs to be read as "12 divided by 3". It is important for children to realise that 12 ÷ 3 is not the same as 3 ÷ 12. Expressions such as "12 divided into 3" are confusing and should be avoided.'

We use the symbol to mean (in this example) 12 ÷ 3 'how many 3s go into 12?'

When the children are introduced to the symbol ÷ it must be explained as above.

Children should be taught the corresponding division facts alongside a new multiplication fact. Children will see the relationship between multiplication and division.

Using the symbol to record

12 ÷ 3 = 4
4 = 12 ÷ 3
12 ÷ ? = 4
12 ÷ 3 = ?
? ÷ 3 = 4
4 = 12 ÷ ?
4 = ? ÷ 3

At this stage children will understand the use of the symbol in a written calculation and they will understand the relationship between multiplication and division.

Children should be able to draw upon their knowledge of multiplication tables to solve any division that occurs as a direct inverse of a multiplication table up to 10 x 10.

56 ÷ 8 = How many 8s go into 56?
Children use their knowledge of the 8x table.
Children may apply other strategies and tools they know such as 'the inverse of the 8x table is halve, halve and halve again.'

45 ÷ 5 = Use knowledge of 5x tables.

In order to draw upon their knowledge of multiplication tables children may need an interim step to support them.

The following are examples of how children may mentally, practically or even in their own written 'jottings' use their multiplication tables to solve division.

1. Using 'train tracks'

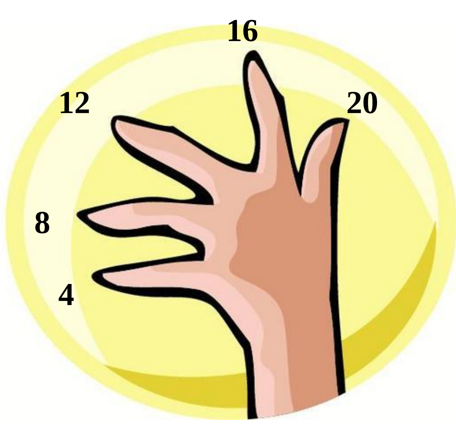
28 ÷ 4 = How many 4s go into 28? Use knowledge of 4x table.

4	8	12	16	20	24	28

Children count up in steps of 4 using their 4x table to reach the target. This supports division learning when the number is divisible exactly by the divisor.

2. Mental strategy and can also be used to record on prepared hands

24 ÷ 4 = How many 4s go into 24? Use knowledge of 4x table.



Children must be able to solve the division calculations using knowledge of multiplication tables for numbers in tables up to 10 x 10.

When the numbers increase beyond each multiplication total eg 10 x 2, 10 x 6 etc… children need other strategies of solving and written methods of recording.

Introduce remainders

29 ÷ 4 = How many 4s go into 29? Count up in 4s to 28 then 1 remaining.

r1

4	8	12	16	20	24	28	~~32~~

Introducing 'chunking' in the form of an informal jotting
Examples

48 ÷ 4 =

Children counting up 'chunks' fours to reach 48

10 x 4 = 40 40
2 x 4 = 8 48

Adding up the total number of fours found to make 48 = 12

96 ÷ 4 =
Children counting up 'chunks' fours to reach 96
10 x 4 = 40 40
10 x 4 = 40 80
4 x 4 = 16 96
Adding up the total number of fours found to make 96 = 24 As division is repeated subtraction (taking away the same number many times) we need to explain to the children we are subtracting one or more 'chunks' of the divisor at a time. The children will be able to see and understand that taking away chunks of the divisor one at a time will achieve the answer.

Formal 'Chunking'

```
                          TU
48 ÷ 4 = 12               48
              10 x 4 =  - 40
                           8
               2 x 4 =  -  8
                           0
```

The number of chunks being subtracted are shaded in grey. These are written first to show how many of the divisor have been taken away. These are then added to find the total number of '4s' in 48.

```
                         TU
96 ÷ 4 = 24              96
             10 x 4 =  - 40
                         56
             10 x 4 =  - 40
                         16
              4 x 4 =  - 16
                          0
```

The number of chunks being subtracted are shaded in grey. These are written first to show how many of the divisor have been taken away. These are then added to find the total number of '4s' in 96.

```
                         TU
96 ÷ 4 = 24              96
             20 x 4 =  - 80
                         16
              4 x 4 =  - 16
                          0
```

The number of chunks being subtracted are shaded in grey. These are written first to show how many of the divisor have been taken away. These are then added to find the total number of '4s' in 96.

This formal chunking would operate in the same way when using numbers where there would be a remainder.

Example of chunking with remainders and when dividing by a two-digit divisor:-

```
324 ÷ 15 = 21 r9          HTU
                          324
             10 x 15 =  - 150
                          174
             10 x 15 =  - 150
                           24
              1 x 15 =  -  15
                            9
```

Introducing other signs and symbols associated with division

$\overline{)\quad}$ used for final stage of chunking and with short division.

Example (not just to be used for division with remainders – as soon as children understand 'chunking' as repeated subtraction, introduce this symbol in the setting out of chunking)

196 ÷ 6 = 32 r 4

```
                 H  T U
                      32 r4
                 6)1 9 6
10 x 6  =         -  6 0
                   1 3 6
10 x 6  =         -  6 0
                     7 6
10 x 6  =         -  6 0
                     1 6
 2 x 6  =          - 1 2
                       4
```

When the children are completely confident with the repeated addition chunking method, they will be shown the compact method. Children must be extremely secure with 'place value' and with their times tables facts before being introduced to this method.

This method is purely used for efficiency and loses the idea of place value that has previously been the focus.

146 ÷ 4

```
         3  6 . 5
       ___________
      |  1  2    2
   4  |  1  4  6 . 0
```

666 ÷ 9

```
           7  4
       ___________
      |    6  3
   9  |  6  6  6
```

967 ÷ 8

```
     1 2 0 . 8 7 5
   ________________
      1     7 6 4
8 )  9 6 7 . 0 0 0
```

Guidelines for compact method

- Set out the calculation with the divisor on the outside and the number being divided underneath .
- Begin by seeing how many times the divisor goes into the most significant number, eg:

146 ÷ 4

```
       3
   _________
     1 2
4 )  1 4 6
```

4 cannot 'go into 1', so the 1 is carried over to the 4 to make it 14. 4 'goes into 14'

3 times (recorded above the division)‾‾‾‾‾ sign) . The remainder of 2 is carried across to the 6 to make it 26.

- Now, work out how many 4s 'go into 26. The answer is 6 (which is recorded above the bus stop). However, there is a remainder of 2. As we need to divide further, a decimal point is added to the original number, the remainder 2 can now be carried over.

```
       3 4
   ___________
     1 2   2
4 )  1 4 6 . 0
```

- Finally, we see how many 4s 'go into 20. The answer is 5 (with no remainder) so this can be added above the division sign.

```
       3 4 . 5
   ___________
     1 2   2
4 )  1 4 6 . 0
```

Don't forget to bring the decimal point up.

Long division

2752 ÷13 =

Start with the digit with the highest place value, here it's the thousands. 13 doesn't go into 2, so the answer won't have any thousands. Exchange 2 thousands for 20 hundreds.

```
    ______
13 ) 2752
```

Next look at the hundreds. 13 goes into 27 twice so put a 2 on the answer line above the 7 in the hundreds column. Subtract 26 from 27 to get a remainder and write it in the hundreds column.

```
                 2
              ______
          13 ) 2752
2 x 13        - 26
               ___
                 1
```

Exchange the remainder from the hundreds into 10 tens and carry the 5 tens down next to the remainder1 to get 15 tens. 13 goes into 15 once so write 1 on the answer line. Subtract 13 to find the remainder and write in the tens column.

```
                 21
              ______
          13 ) 2752
2 x 13 =      - 26
               ___
                 15
1 x 13   =     - 13
                ___
                  2
```

Finally look at the units. Exchange the 2 tens into 20 units and carry the 2 units down to get 22 units. 13 into 22 goes once so write 1 on the number line and subtract 13 to find the remainder as there are no more digits to carry down.

```
                 211 r 9
              ______
          13 ) 2752
2 x 13 =      - 26
               ___
                 15
1 x 13 =       - 13
                ___
                  22
1 x 13   =      - 13
                 ___
                   9 – remainder
```

The answer is 211 remainder 9.

Division of fractions

Dividing a fraction by a whole number

Multiply the denominator by the whole number and the numerator doesn't change.

$$\frac{1}{2} \div 2 = \frac{1}{2\times2} = \frac{1}{4}$$

Dividing a fraction by another fraction

$$\frac{1}{2} \div \frac{1}{6}$$

Turn the second fraction upside down (it becomes a **reciprocal**):

$$\frac{1}{6} \text{ becomes } \frac{6}{1}$$

Multiply the first fraction by the recipocal

$$\frac{1}{2} \times \frac{6}{1} = \frac{1 \times 6}{2 \times 1} = \frac{6}{2}$$

Simplify the fraction:

$$\frac{6}{2} = 3$$